OCR
Oxford Cambridge and RSA

Thursday 14 May 2015 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $x+\mathrm{i}y$ is denoted by z. Express $3zz^*-|z|^2$ in terms of x and y. **[3]**

2 Find $\sum_{r=1}^{n}(3r^2-5)$, expressing your answer in a fully factorised form. **[4]**

3 The matrix $\mathbf{A}$ is given by $\mathbf{A}=\begin{pmatrix}2 & a\\ 0 & 1\end{pmatrix}$, where a is a constant.

(i) Find $\mathbf{A}^{-1}$. **[2]**

The matrix $\mathbf{B}$ is given by $\mathbf{B}=\begin{pmatrix}2 & a\\ 4 & 1\end{pmatrix}$.

(ii) Given that $\mathbf{PA}=\mathbf{B}$, find the matrix $\mathbf{P}$. **[3]**

4 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(3r+1)=n(n+1)^2$. **[5]**

5 The loci C_1 and C_2 are given by $|z+2|=2$ and $\arg(z+2)=\frac{5}{6}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[4]**

(ii) Find the complex number represented by the intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z+2| \leqslant 2 \text{ and } \tfrac{5}{6}\pi \leqslant \arg(z+2) \leqslant \pi. \quad \textbf{[2]}$$

6 The matrix $\mathbf{M}$ is given by $\mathbf{M}=\begin{pmatrix}0 & 2\\ -1 & 0\end{pmatrix}$.

(i) The diagram in the Printed Answer Book shows the unit square $OABC$. The image of the unit square under the transformation represented by $\mathbf{M}$ is $OA'B'C'$. Draw and label $OA'B'C'$, indicating clearly the coordinates of A', B' and C'. **[3]**

(ii) The transformation represented by $\mathbf{M}$ is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a possible pair of transformations P and Q and state the matrices that represent them. **[4]**

7 **(i)** Use an algebraic method to find the square roots of the complex number $5+12\mathrm{i}$. You must show sufficient working to justify your answers. **[5]**

(ii) Hence solve the quadratic equation $x^2-4x-1-12\mathrm{i}=0$. **[5]**

8 **(i)** Show that $\frac{3}{r-1}-\frac{2}{r}-\frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=2}^{n}\frac{4r+2}{r(r^2-1)}$. **[6]**

(iii) Hence find the value of $\sum_{r=4}^{\infty}\frac{4r+2}{r(r^2-1)}$. **[2]**

9 The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

(i) Find the values of a for which **D** is singular. **[6]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 2x + ay + 3z &= 2 \\ y + az &= 0 \end{aligned}$$

For each of the following values of a, determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 1$

[4]

10 The cubic equation $x^3 + 4x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u. **[3]**

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$. **[7]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$z^* = x - iy$ $\|z\| = \sqrt{x^2 + y^2}$ $2(x^2 + y^2)$	B1 B1 B1 **[3]**	Conjugate stated or used Modulus or it's square stated or used Obtain correct answer, a.e.f. but not involving i	
2			½ $n(n + 1)(2n + 1) - 5n$ ½ $n(2n - 3)(n + 3)$ or $n(n - \frac{3}{2})(n + 3)$	M1* A1 DM1 A1 **[4]**	Express as difference using standard result for $\sum r^2$ Correct unsimplified expression Obtain at least factor of n Obtain correct answer, only these versions	
3	(i)		$\frac{1}{2}\begin{pmatrix} 1 & -a \\ 0 & 2 \end{pmatrix}$ or equivalent	B1 B1 **[2]**	Both diagonals correct Divide by correct determinant	
3	(ii)	***Either***	**P** = **BA**$^{-1}$ $\begin{pmatrix} 1 & 0 \\ 2 & 1-2a \end{pmatrix}$	B1 M1 A1ft **[3]**	State or use correct expression for **P** Multiplication attempt, 2 elements correct for any pair of matrices Obtain correct answer a.e.f. ft for their **(i)**	
		Or	Using **PA** = **B**	B1 M1 A1	State or find correct 1st column of **P** Multiplication attempt to find "1 – 2a " Obtain completely correct answer	
4			$k(k+1)^2 + (k+1)(3k+4)$ $(k+1)(k+2)^2$	B1 M1* DM1 A1 B1 **[5]**	Show sufficient working to verify result true when $n = 1$ Add next term in series Attempt to factorise their expression Sufficient working to obtain this correct answer Clear statement of induction process, provided previous 4 marks earned	

Question			Answer	Marks	Guidance	
5	(i)			B1 B1 B1 B1 **[4]**	Circle centre (–2, 0) or circle centre (2, 0) Touching y-axis at origin Half line with negative slope upwards Completely correct diagram	
5	(ii)		$-2-\sqrt{3}+i$	B1ft B1ft **[2]**	Correct real part and correct imaginary part of a complex number, ft for their half line from centre of their circle, allow decimals (-3.73 or better) or trig expressions	
5	(iii)			B1ft B1 **[2]**	Shade inside their circle Completely correct diagram and shading **S.C. allow last B1 for radius or complete line**	
6	(i)		$A'(0,-1)\quad B'(2,-1)\quad C'(2,0)$	B1 B1 B1 **[3]**	Coordinates of any 2 images seen Coordinates of 3rd image seen Completely correct labelled diagram, must include indication of coordinates	Might be columns
6	(ii)		$\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ and $\begin{pmatrix}2&0\\0&1\end{pmatrix}$ Or $\begin{pmatrix}1&0\\0&2\end{pmatrix}$ and $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$	B1 B1 B1ft B1ft **[4]**	Rotation and stretch or vice versa Rotation 90° clockwise, then Stretch s.f. 2 parallel to x-axis Or Stretch s.f. 2 parallel to y-axis & Rotation 90° clockwise Correct matrix, Correct matrix **S.C. If 1 matrix correct, correct 2nd matrix can be found by matrix multiplication and not be necessarily consistent with their transformation, but not ft.**	Must be a correct pair in correct order Consistent with their pair of transformations Just a trig form for rotation not acceptable

Question			Answer	Marks	Guidance	
7	(i)			M1	Attempt to equate real and imaginary parts of $(x + iy)$ and $5 + 12i$	
			$x^2 - y^2 = 5, 2xyi = 12i$	A1	Obtain both results or equivalent	
				M1	Obtain and solve a quadratic in x^2 or y^2 or solve by inspection	
			$3 + 2i$ and $-3 - 2i$ or $\pm(3 + 2i)$	A1 A1	Obtain correct answers as complex numbers	
				[5]	**S.C. $\pm(3 \pm 2i)$ scores A1**	
7	(ii)			M1	Solve using quadratic formula or complete square	
			$(4 \pm 2\sqrt{5 + 12i})/2$	A1	Obtain correct answers, or simpler version	
				M1	Use result(s) from **(i)**	
			$5 + 2i$ and $-1 - 2i$ or $2 \pm (3 + 2i)$	A1 A1	Obtain correct answers	If more than 2 roots A0 A0
				[5]		
8	(i)			M1	Use correct common denominator, numerator must be quadratic	
				A1	Obtain **given** result	
				[2]		
8	(ii)			M1	Express terms as differences using **(i)**	
				M1	Attempt this for at least first 3 terms	
				A1	First 3 terms all correct	
				A1	Last 2 terms correct	
			$\frac{7}{2} - \frac{3}{n} - \frac{1}{n+1}$	M1	Show terms cancelling	Need not be tidied up
				A1	Obtain correct answer, must be in terms of n	
				[6]		
8	(iii)		$\frac{5}{4}$	M1	Attempt to start summation at correct term	Could be $\Sigma_2^{\infty} - \Sigma_2^3$
				A1	Obtain correct answer from correct working	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)			M1 M1 M1 A1 M1 A1 **[6]**	Attempt to find det **D** Show correct process for a 3 × 3, condone sign errors Show correct processes for a 2 × 2 Obtain correct answer Attempt to solve det **D** = 0 Obtain correct answers	Or Cramer's rule or similar
			$a^2 - 6a + 5$ $a = 5$ or 1			
9	(ii)	(a)(b)		B1 B1 M1 A1 **[4]**	State unique solution State non unique solutions Attempt to solve equations with $a = 1$ Explain inconsistency with correct working **S.C. Answer to (i) wrong, allow correct unique/non-unique B1ft, B1ft only**	

Question			Answer	Marks	Guidance	
10	(i)			B1	Use given substitution correctly in LHS of equation	
				M1	Rearrange and square to eliminate $\sqrt{u}$ or multiply by $u^{\frac{3}{2}} + 4u^{\frac{1}{2}} - 3$	
			$u^3 + 8u^2 + 16u - 9 = 0$	A1	Obtain correct answer, must be an equation = 0	
				[3]		
10	(ii)	***Either***	$\alpha\beta\gamma = -3$	B1	State or use correct result	
			$\sum\alpha^2 = -8 \qquad \sum\alpha^2\beta^2 = 16$	B1B1	Use correct result, using correct (i) or using an identity involving $\sum\alpha = 0, \sum\alpha\beta = 4$	
			$(\sum\alpha^2)^2 = \sum\alpha^4 + 2\sum\alpha^2\beta^2$	M1*	Obtain an identity connecting $\sum\alpha^4$ and $(\sum\alpha^2)^2$	
				A1	Obtain a correct answer	
				DM1	Use their values in their expression	
			29	A1	Obtain correct answer, c.w.o.	
				[7]		
		Or	$\alpha\beta\gamma = -3$	B1	State or use correct result	
			$\sum\alpha = 0, \sum\alpha\beta = 4, \sum\alpha^2 = -8,$ $\sum\alpha^2\beta^2 = 16$	B1 B1	Use any 2 correct B1, other 2 correct B1	
				M1	Expand $(\alpha+\beta+\gamma)^4$ and get expression involving symmetric functions only	
			$\sum\alpha^4 + 4\sum\alpha^2.\sum\alpha\beta + 6\sum\alpha^2\beta^2 + 8\alpha\beta\gamma\sum\alpha$	A1	Obtain correct expression	
				M1	Use their values in their expression	
				A1	Obtain correct answer, c.w.o.	